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Bayesian estimation of the mean of exponential distribution using ranked set sampling with unequal samples

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ABSTRACT
In this paper, we study Bayes estimators for the mean of the exponential distribution based on ranked set sample with unequal samples (RSSU) proposed and studied by Bhoj (2001). We obtain the Bayes estimates of the scale parameter using both the squared error loss (SEL) function and the linear exponential (LINEX) loss function. Under the assumption of gamma and Jeffreys prior distributions for the scale parameter, we obtain the Bayes estimators. We compare different estimators through simulations for illustration and compute the bias and mean squared error (MSE) of these estimators. We observe that the proposed estimators based on RSSU are more efficient than those based on SRS when the scale parameter follows the Jeffreys prior distribution compared to when it follows the gamma prior distribution.

KEYWORDS
Bayes estimation; Bias; Conjugate prior; Jeffreys prior; Mean squared error (MSE); Posterior distribution;, Ranked set sampling with unequal samples (RSSU).

1. Introduction

Ranked set sampling (RSS) is a method of sampling that can be advantageous when quantification of all sampling units is costly but a small set of units can be easily ranked according to the character under investigation without actual quantification. In these scenarios, the researchers frequently have relevant auxiliary information that they can use to predict the ranks of the units in the set. These indicated ranks allow researchers to select which units from a set to include in a full measurement, likely to result in a sample that is usually more informative than one obtained from a simple random sample. To obtain a ranked set sample of size $n = mk$, the selection process involves drawing m random samples, with m units in each sample, and ranking the units in each sample using methods that do not require actual measurements. The unit with the lowest rank in the first sample is then measured, followed by the unit with the second lowest rank in the second sample, and so on, until the unit with the highest rank in the last sample is measured. This constitutes one cycle of the ranked set sample of size m, and the cycle is repeated k times to obtain a sample of size n.

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Typically, the value of m is chosen between two and five to ensure accurate ranking of the units in each set.

There is frequently some prior knowledge about the parameter that can be used to obtain a better estimate when estimating an unknown parameter. One well-known method for incorporating this prior knowledge is the Bayesian approach, which utilizes a prior distribution to represent the available information about the parameter.

The utilization of Bayesian estimation with Ranked Set Sampling (RSS) was first explored by Al-Saleh and Muttlak (1998), where they investigated Bayes estimators for exponential and normal distributions. Kim and Arnold (1999) employed the Gibbs sampler to study Bayesian parameter estimation for both balanced and generalized RSS. Lavin (1999) work examined the optimality of RSS procedures from a Bayesian perspective. Al-Saleh and Muttlak (2000) studied the Bayesian estimation of exponential distribution using RSS with a conjugate prior, and applied it to real-world data. Sadek et al. (2015) obtained Bayes estimators of exponential distributions based on RSS under a LINEX loss function. Mohie El-Din et al.(2015) investigated Bayes estimation and prediction for Pareto distributions based on RSS. Sanku Dey et al. (2017) addressed the problem of Bayesian parameter estimation for Rayleigh distributions using various ranked set sampling schemes.

In recent years, several modifications have been proposed for the classical Ranked Set Sampling (RSS) method with unequal sample sizes. Bhoj (2001), Al-Odat and Al-Saleh (2001), and Biradar and Santosha (2014) have suggested important modifications such as Ranked Set Sampling with Unequal Samples (RSSU), Moving Ranked Set Samples (MERSS), and Maximum Ranked Set Sampling with Unequal Samples (MaxRSSU). Al-Hadhramia Al.Omari (2009) studied the Bayesian inference of the variance of the normal distribution using MERSS, while Al-Hadhrami and Al-Omari(2012) focused on Bayesian estimation of the mean of the normal distribution using MERSS. Biradar and Shivanna (2016) obtained Bayes estimators based on MaxRSSU. In many applications, ranked set sampling with unequal sample sizes arises naturally, such as with commuters on different public buses or patients waiting in doctors' waiting rooms with varying set sizes.

Bhoj(2001) proposed a ranked set sampling procedure with unequal set sizes (RSSU) to estimate the population mean, and showed that the estimators based on RSSU are more efficient than the estimators based on SRS, RSS and median ranked set sampling (MRSS, see Muttlak 1997) when the distributions under considerations are symmetrical or moderately skewed. In RSSU, we draw m samples, where the size of the i-th sample is $2i-1$, for $i = 1, 2, ..., m$. The steps in RSSU are the same as in RSS. In both sampling procedures, we measure accurately only m observations. However, in RSSU, we rank only m^2-1 observations. When m is even, half the set sizes are less than m and the other half are greater than m . In the case of odd m , one sample is of set size m, $(m-1)/2$ samples are smaller than m, and the other $(m-1)/2$ samples are greater than m. This process is repeated k times in order to get a RSSU of size $n = mk$. Dong et al.(2013) used RSSU to estimate the reliability $P(X > Y)$ for a system with strength X and stress Y both following exponential distributions. Zhang et al.(2014) proposed sign test based on RSSU. They have provided weighted sign test and it is shown that optimal weighted sign test under RSSU is more efficient than optimal sign test under RSS and MRSS. Recently, Biradar (2022) developed maximum likelihood estimators for the location-scale family of distributions based on RSSU. It is shown that asymptotic efficiencies of the MLE based on RSSU are considerably better than those of the estimators based on RSS with the same number of observations. As far as we know, no Bayes estimators based on RSSU has been studied, therefore, our main objective of this study is to develop Bayes estimators based on RSSU and explore various properties.

The article is organized as follows. In Section 2, we present the general approach for Bayesian estimation using the RSSU method. Section 3 examines the application of this method to estimate the scale parameter of the exponential distribution using SRS. Section 4 explores the use of RSSU for the same distribution. In Section 5, we evaluate the performance of proposed estimator by conducting simulations and making comparisons.

2. General Setup

Let $X_1, X_2, \ldots, X_m$ be independent and identically distributed (iid) random variables with pdf $f(x|\theta)$ and cdf $F(x|\theta)$, where θ is the parameter and F is known. Let $\{X_{i1}, X_{i2}, \ldots, X_{i2i-1}\}$, i=1,2,...,m, be m sets of SRS with size of the i-th sample is $2i-1, i = 1, 2, ..., m$, from the same distribution. Let $X_{i:2i-1}$ = ith order statistic$\{X_{i1}, \ldots, X_{i2i-1}\}$, for $i = 1, 2, ..., m$.

Then $\mathbf{X_{RSSU}} = \{X_{i:2i-1}; i = 1, .., m\}$, constitute a RSSU of size m (see Bhoj, 2001). For simplicity take $X_{i:2i-1} = Y_i$, for $i = 1, 2, ..., m$. If the judgment ranking is perfect, then Y_i's are independent and Y_i has the same pdf as the i-th order statistic of a SRS of size $2i-1$ from $f(y|\theta)$, and is given by

$$f_{i:2i-1}(y|\theta) = \frac{1}{B(i,i)} \left[F\left(y|\theta\right)\right]^{i-1} \left[1 - F\left(y|\theta\right)\right]^{i-1} f\left(y|\theta\right), \tag{1}$$

and cdf

$$F_{i:2i-1}(y|\theta) = \sum_{j=i}^{2i-1} \binom{2i-1}{j} F^j(y|\theta)[1 - F(y|\theta)]^{2i-1-j}.$$

Let $Y_1, \ldots, Y_m$ be a RSSU of size m from the distribution with pdf (1) then the joint distribution of the RSSU sample is

$$f(\underline{y}|\theta) = \prod_{i=1}^{m} \frac{1}{B(i,i)} \left[F\left(y_i|\theta\right)\right]^{i-1} \left[1 - F\left(y_i|\theta\right)\right]^{i-1} f\left(y_i|\theta\right). \tag{2}$$

In order to study Bayes estimators based on SRS and RSSU we consider conjugate and Jeffreys priors. Let $g(\theta)$ and $\pi(\theta|\underline{y})$ denote respectively prior and posterior distributions. The posterior distribution of θ given a RSSU sample $\underline{y} = (y_1, \ldots, y_m)$ is given by

$$\pi(\theta|\underline{y}) = \frac{f(\underline{y}|\theta)g(\theta)}{\int_\theta f(\underline{y}|\theta)g(\theta)d\theta} \tag{3}$$

Squared error loss function (SEL) is a commonly used method for evaluating the performance of an estimator in Bayesian estimation, as observed in various studies for example Box and Tiao (1973) and Berger (1985). The squared error loss function (SEL) treats overestimation and underestimation equally due to its symmetric nature. The Bayes estimator of θ based on the SEL function using RSSU data is

$$\hat{\theta}_{Sel}(\underline{y}) = \frac{\int_\theta \theta f(\underline{y}|\theta)g(\theta)d\theta}{\int_\theta f(\underline{y}|\theta)g(\theta)d\theta}. \tag{4}$$

It should be noted that the use of the squared error loss function (SEL) is appropriate only when the losses are symmetric. For instance, in the case of estimating the survival function, the symmetric loss function may not be suitable. Therefore, asymmetric loss functions have been explored in the literature. One of the commonly used asymmetric loss functions is the linear exponential (LINEX) loss, which is a natural extension of SEL. This loss function was introduced by Varian (1975) and was popularized by Zellner (1986). The LINEX loss function for the parameter θ can be expressed as

$$L(\Delta, c) = d(e^{c\Delta} - c\Delta - 1), \tag{5}$$

where $\Delta = (\hat{\theta} - \theta)$, and $\hat{\theta}$ is an estimate of θ and $c \neq 0$, c and d are shape and scale parameters. The sign and magnitude of the shape parameter c represent the direction and degree of symmetry, respectively. When c is close to zero the LINEX loss function is approximately squared error loss function. The Bayes estimator of θ based on the LINEX loss function using RSSU data is

$$\hat{\theta}^J_{Lnx}(\underline{y}) = -\frac{1}{c} lnE(e^{-c\theta}), \tag{6}$$

provided the expectation exists.

3. Bayes Estimator of the scale parameter of exponential distribution based on SRS

In this section, we obtain the Bayes estimates of the scale parameter of exponential distribution based on SRS. The pdf of the exponential distribution is given by

$$f(x|\theta) = \theta e^{-\theta x}, \quad x > 0, \theta > 0, \tag{7}$$

and its distribution function is

$$F(x|\theta) = 1 - e^{-\theta x}, x > 0, \theta > 0. \tag{8}$$

To derive a Bayesian estimator of the unknown parameter of the exponential distribution, it is assumed that the parameter θ follows the gamma prior distribution with density

$$g(\theta) \propto \theta^{\alpha-1} e^{-\theta\beta}, \;\; 0 < \theta < \infty. \tag{9}$$

The gamma prior distribution is highly flexible as it can accommodate a wide range of shapes depending on the values of its hyperparameters. Thus, the family of gamma prior can be used as a suitable prior for θ. Note that non-informative prior is a special case of gamma priors and can be obtained by letting $\alpha = \beta = 0$. Then $g(\theta)$ becomes the Jeffreys prior which is given by

$$g(\theta) \propto \frac{1}{\theta}, \quad \theta > 0. \tag{10}$$

The posterior distribution of the parameter θ given SRS $\underline{x} = (x_1, \ldots, x_m)$ is given by

$$\pi(\theta|\underline{x}) = \frac{\theta^{m+\alpha-1} e^{-\theta(\sum_{i=1}^{m} x_i + \beta)}}{\Gamma(m+\alpha)(\sum_{i=1}^{m} x_i + \beta)^{-(m+\alpha)}} \; . \tag{11}$$

Thus, the Bayes estimator of θ under the SEL function using SRS is

$$\hat{\theta}_{Sel}(\underline{x}) = \frac{m+\alpha}{\sum_{i=1}^{m} x_i + \beta}. \tag{12}$$

The Bayes estimate of θ under the LINEX loss function using SRS is given by

$$\hat{\theta}_{Lnx}(\underline{x}) = \frac{m+\alpha}{c} ln\left(1 + \frac{c}{n\bar{x}\beta}\right). \tag{13}$$

Suppose θ follows non-informative prior distribution given by (10), then the posterior distribution of θ is given by

$$\pi(\theta|\underline{x}) = \frac{(n\bar{x})^n \theta^{n-1} e^{-\theta n\bar{x}}}{\Gamma(n)} \tag{14}$$

Thus, the Bayes estimators of θ under SEL and LINEX loss functions, respectively, based on SRS are

$$\hat{\theta}^J_{Sel}(\underline{x}) = \frac{1}{\bar{x}} \tag{15}$$

and

$$\hat{\theta}^J_{Lnx}(\underline{x}) = \frac{m}{c} ln\left(1 + \frac{c}{n\bar{x}}\right). \tag{16}$$

4. Bayes estimator of the scale parameter of exponential distribution based on RSSU

The following subsections present the Bayes estimator for the scale parameter θ of the exponential distribution using both SEL and LINEX loss functions based on RSSU. The estimators are obtained based on the assumption that the prior distributions for θ are conjugate prior and Jeffreys prior.

4.1. Bayesian Estimators based on Conjugate prior of θ

Assuming that the random variable X has an exponential distribution, with pdf and cdf given by (7) and (8), respectively. From equation (1), for the exponential distribution the pdf of jth order statistic of a random sample of size $2j-1$ is given by

$$f(y_j|\theta) = \theta \sum_{k=0}^{j-1} a_k(j) e^{-(j+k)\theta y_j}, y_j > 0, \theta > 0, \tag{17}$$

where $a_k(j) = \frac{1}{B(j,j)}(-1)^k$.
From (2) the joint pdf of the RSSU sample is given by

$$f(\underline{y}|\theta) = \sum_{i_1=0}^{0} \sum_{i_2=0}^{1} \cdots \sum_{i_m=0}^{m-1} \left[\prod_{j=1}^{m} a_{i_j}(j) \right] \theta^m e^{-\theta \sum_{j=1}^{m} y_j(i_j+j)}, \ \ \theta > 0. \tag{18}$$

Combining the joint pdf (18) and gamma prior density (9) the posterior pdf of θ given the RSSU sample of size m simplifies to

$$\pi(\theta|\underline{y}) = \frac{\theta^{m+\alpha-1} \sum_{i_1=0}^{0} \sum_{i_2=0}^{1} \cdots \sum_{i_m=0}^{m-1} (\prod_{j=1}^{m} a_{i_j}(j)) e^{-\theta(\sum_{j=1}^{m} y_j(i_j+j)+\beta)}}{\sum_{i_1=0}^{0} \sum_{i_2=0}^{1} \cdots \sum_{i_m=0}^{m-1} (\prod_{j=1}^{m} a_{i_j}(j)) \Gamma(m+\alpha) [\sum_{j=1}^{m} y_j(i_j+j)+\beta]^{-(m+\alpha)}}. \tag{19}$$

Thus, the Bayes estimator of θ based on RSSU under the SEL function is given by

$$\begin{aligned} \hat{\theta}_{Sel}(\underline{y}) &= \int_0^\infty \theta \pi(\theta|\underline{y}) d\theta \\ &= \frac{(m+\alpha) \sum_{i_1=0}^{0} \sum_{i_2=0}^{1} \cdots \sum_{i_m=0}^{m-1} (\prod_{j=1}^{m} a_{i_j}(j)) (\sum_{j=1}^{m} y_j(i_j+j)+\beta)^{-(m+\alpha+1)}}{\sum_{i_1=0}^{0} \sum_{i_2=0}^{1} \cdots \sum_{i_m=0}^{m-1} (\prod_{j=1}^{m} a_{i_j}(j)) [\sum_{j=1}^{m} y_j(i_j+j)+\beta]^{-(m+\alpha)}}. \end{aligned} \tag{20}$$

In order to obtain the Bayes estimator of θ based on RSSU using the LINEX loss function, we find that

$$E(e^{-c\theta}) = \frac{\sum_{i_1=0}^{0} \sum_{i_2=0}^{1} \cdots \sum_{i_m=0}^{m-1} (\prod_{j=1}^{m} a_{i_j}(j)) (\sum_{j=1}^{m} y_j(i_j+j)+\beta+c)^{-(m+\alpha)}}{\sum_{i_1=0}^{0} \sum_{i_2=0}^{1} \cdots \sum_{i_m=0}^{m-1} (\prod_{j=1}^{m} a_{i_j}(j)) [\sum_{j=1}^{m} y_j(i_j+j)+\beta]^{-(m+\alpha)}}. \tag{21}$$

Now substituting (21) into (6), we obtain the Bayes estimator of θ under the LINEX loss function and is denoted by $\hat{\theta}_{Lnx}(\underline{y})$.

4.2. Bayesian Estimators based on Non-informative prior of θ

The posterior density of θ given a RSSU, when θ has non-informative prior distribution given by (10) simplifies to

$$\pi(\theta|\underline{y}) = \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_m=0}^{m-1}(\prod_{j=1}^{m}a_{i_j}(j))\theta^{m-1}e^{-\theta\sum_{j=1}^{m}y_j(i_j+j)}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_m=0}^{m-1}(\prod_{j=1}^{m}a_{i_j}(j))\Gamma m[\sum_{j=1}^{m}y_j(i_j+j)]^{-m}}. \tag{22}$$

Therefore, given the non-informative prior the Bayes estimator under the SEL function becomes

$$\hat{\theta}_{Sel}^{J}(\underline{y}) = \frac{m\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_m=0}^{m-1}(\prod_{j=1}^{m}a_{i_j}(j))(\sum_{j=1}^{m}y_j(i_j+j))^{-(m+1)}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_m=0}^{m-1}(\prod_{j=1}^{m}a_{i_j}(j))[\sum_{j=1}^{m}y_j(i_j+1)]^{-m}}. \tag{23}$$

In order to obtain the Bayes estimator of θ under the LINEX loss function, we find that

$$E(e^{-c\theta}) = \frac{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_m=0}^{m-1}(\prod_{j=1}^{m}a_{i_j}(j))(\sum_{j=1}^{m}y_j(i_j+j)+c)^{-(m)}}{\sum_{i_1=0}^{0}\sum_{i_2=0}^{1}\cdots\sum_{i_m=0}^{m-1}(\prod_{j=1}^{m}a_{i_j}(j)[\sum_{j=1}^{m}y_j(i_j+j)]^{-m}}. \tag{24}$$

Now substituting (24) into (6), we obtain the Bayes estimator $\hat{\theta}_{Lnx}^{J}(\underline{y})$ under the LINEX loss function.

5. Simulation study

We performed a simulation study using R-language version 3.1.1 to compare the performance of several Bayes estimators for the scale parameter of the exponential distribution that were developed in this paper. We computed the bias and mean squared error (MSE) for all four estimators based on simple random sampling (SRS) and ranked set sampling with unequal samples (RSSU). The results of the simulation are presented in Table 1 and Table 2, respectively. Additionally, in Table 3, we compared the relative efficiency of the Bayes estimators based on RSSU to those based on SRS using numerical results. The relative efficiencies of the Bayes estimator based on RSSU compared to those based on SRS are defined by

$$RE_{J,Sel} = \frac{MSE(\hat{\theta}_{Sel}^{J}(\underline{x}))}{MSE(\hat{\theta}_{Sel}^{J}(\underline{y}))} \tag{25}$$

$$RE_{G,Sel} = \frac{MSE(\hat{\theta}_{Sel}(\underline{x}))}{MSE(\hat{\theta}_{Sel}(\underline{y}))}, \tag{26}$$

$$RE_{J,Lnx} = \frac{MSE(\hat{\theta}_{Lnx}^{J}(\underline{x}))}{MSE(\hat{\theta}_{Lnx}^{J}(\underline{y}))} \tag{27}$$

and

$$RE_{G,Lnx} = \frac{MSE(\hat{\theta}_{Lnx}(\underline{x}))}{MSE(\hat{\theta}_{Lnx}(\underline{y}))}, \tag{28}$$

where suffix J denotes Jeffreys prior and suffix G denotes gamma prior, respectively.

Table 1. *Bias of the Bayes estimators based on SRS and RSSU when θ=2, $\alpha = \beta = 1$ and $c = 1, -1$ for $m = 2(1)5$*

	Bias($\hat{\theta}^J_{Sel}$)		**Bias($\hat{\theta}_{Sel}$)**			**Bias($\hat{\theta}^J_{Lnx}$)**		**Bias($\hat{\theta}_{Lnx}$)**	
	Jeffrey prior		**Gamma prior**			**Jeffrey prior**		**Gamma prior**	
m	**SRS**	**RSSU**	**SRS**	**RSSU**	**c**	**SRS**	**RSSU**	**SRS**	**RSSU**
2	0.4071	0.2443	0.2863	0.2322	1	0.1545	0.1312	0.1726	0.1584
					-1	0.5057	0.4181	0.4818	0.3460
3	0.2218	0.0807	0.2151	0.1087	1	0.1150	0.0488	0.1450	0.0794
					-1	0.3542	0.1199	0.3200	0.1432
4	0.1510	0.0426	0.1696	0.0634	1	0.0953	0.0286	0.1278	0.0499
					-1	0.2565	0.0634	0.2441	0.0834
5	0.1196	0.0271	0.1404	0.0422	1	0.0600	0.0170	0.0924	0.0322
					-1	0.1568	0.0378	0.1720	0.0528

Table 2. *MSE of the Bayes estimators based on SRS and RSSU when θ=2, $\alpha = \beta = 1$ and $c = 1, -1$ for $m = 2(1)5$*

	MSE($\hat{\theta}^J_{Sel}$)		**MSE($\hat{\theta}_{Sel}$)**			**MSE($\hat{\theta}^J_{Lnx}$)**		**MSE($\hat{\theta}_{Lnx}$)**	
	Jeffrey prior		**Gamma prior**			**Jeffrey prior**		**Gamma prior**	
m	**SRS**	**RSSU**	**SRS**	**RSSU**	**c**	**SRS**	**RSSU**	**SRS**	**RSSU**
2	1.5254	0.4369	0.2794	0.2094	1	0.2897	0.1875	0.1393	0.1255
					-1	1.7435	1.2444	0.5304	0.4249
3	0.4269	0.0753	0.1957	0.0704	1	0.1828	0.0564	0.1143	0.0541
					-1	1.0034	0.1067	0.4040	0.0947
4	0.2138	0.0378	0.1382	0.0379	1	0.1151	0.0317	0.0930	0.0317
					-1	0.4381	0.0458	0.2253	0.0456
5	0.1696	0.0232	0.1145	0.0235	1	0.0773	0.0210	0.0669	0.0211
					-1	0.2051	0.0260	0.1357	0.0264

Table 3. *Relative efficiencies of Bayes estimators when* $\theta = 2, \alpha = \beta = 1$ *and* $c = 1, -1$ *for* $m = 2(1)5$

m	$RE_{J,Sel}$	$RE_{G,Sel}$	c	$RE_{J,Lnx}$	$RE_{G,Lnx}$
2	3.4914	1.3343	1	1.5451	1.1099
			-1	1.4011	1.2483
3	5.6691	2.7810	1	3.2403	2.1136
			-1	9.4014	4.2663
4	5.6561	3.6457	1	3.6268	2.9307
			-1	9.5624	4.9434
5	7.2954	4.8648	1	3.6837	3.1666
			-1	7.8760	5.1314

Table 1 shows that Bayes estimators based on RSSU samples using SEL and LINEX loss functions have a smaller bias compared to the corresponding estimators based on SRS samples when the prior distribution of θ is either a Jeffreys non-informative prior or a gamma prior. Additionally, when the prior distribution follows a Jeffreys prior distribution, the bias of Bayes estimators is small for both sampling schemes under both SEL and LINEX loss functions. Table 2 displays that the mean squared error (MSE) of Bayes estimators based on RSSU under SEL and LINEX loss functions are almost equal when θ follows either a gamma or a Jeffreys prior distribution. Additionally, we can observe that the MSE of Bayes estimators based on SRS is much lower when θ follows a gamma prior distribution compared to the corresponding estimators when θ follows a Jeffreys prior distribution. When $c = 1$, the bias and MSE of the Bayes estimator under the LINEX loss function are smaller than when $c = -1$.

Table 3 reveals that the relative efficiencies of estimators using both SEL and LINEX loss functions are at least 1.12 times higher when θ follows a Jeffreys prior distribution than the corresponding estimators when θ follows a gamma prior distribution. The results in Table 3 suggest that having some prior information can be superior to relying solely on a gamma prior distribution.

6. Summary and conclusions

This paper discusses Bayesian estimators for the mean of the exponential distribution based on SRS and RSSU sampling schemes. Bayesian estimators are derived under both square error and LINEX loss functions using conjugate prior and Jeffreys prior distributions. A simulation study is conducted to compare the different estimators. The Simulation results show that the MSE of Bayes estimators based on RSSU under both SEL and LINEX loss functions are almost the same when θ follows either a gamma prior or a Jeffreys prior. The relative efficiencies of the Bayes estimators using the LINEX loss function when the shape parameter c= -1 are higher than the corresponding estimators using SEL functions. This suggests that asymmetric loss functions may be more useful than squared error loss functions. It would be useful to conduct further studies of other types of asymmetric loss functions.

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